BASIC OLIGOPOLY MODELS
Overview

I. Conditions for Oligopoly?

II. Role of Strategic Interdependence

III. Profit Maximization in Four Oligopoly Settings
   • Sweezy (Kinked-Demand) Model
   • Cournot Model
   • Stackelberg Model
   • Bertrand Model

IV. Contestable Markets
Oligopoly Environment

• Relatively few firms, usually less than 10.
  • Duopoly - two firms
  • Triopoly - three firms
• The products firms offer can be either differentiated or homogeneous.
• Firms’ decisions impact one another.
• Many different strategic variables are modeled:
  • No single oligopoly model.
Role of Strategic Interaction

- Your actions affect the profits of your rivals.
- Your rivals’ actions affect your profits.
- How will rivals respond to your actions?
An Example

- You and another firm sell differentiated products.
- How does the quantity demanded for your product change when you change your price?
D_2 (Rival matches your price change)

D_1 (Rival holds its price constant)
D2 (Rival matches your price change)

Demand if Rivals Match Price Reductions but not Price Increases

(D1) (Rival holds its price constant)
Key Insight

• The effect of a price reduction on the quantity demanded of your product depends upon whether your rivals respond by cutting their prices too!

• The effect of a price increase on the quantity demanded of your product depends upon whether your rivals respond by raising their prices too!

• Strategic interdependence: You aren’t in complete control of your own destiny!
Sweezy (Kinked-Demand) Model

Environment

- Few firms in the market serving many consumers.
- Firms produce differentiated products.
- Barriers to entry.
- Each firm believes rivals will match (or follow) price reductions, but won’t match (or follow) price increases.
- Key feature of Sweezy Model
  - Price-Rigidity.
Sweezy Demand and Marginal Revenue

\[ D_2 \text{ (Rival matches your price change)} \]

\[ D_S \text{: Sweezy Demand} \]

\[ \text{(Rival holds its price constant)} \]

\[ \text{MR}_S \text{: Sweezy MR} \]
Sweezy Profit-Maximizing Decision

- $D_1$: Rival holds price constant
- $D_2$: Rival matches your price change

$D_S$: Sweezy Demand

$P_0$: Equilibrium price

$Q_0$: Equilibrium quantity

$MC_1$, $MC_2$, $MC_3$: Cost curves

$MR_S$: Marginal Revenue of the Sweezy firm
Sweezy Oligopoly Summary

• Firms believe rivals match price cuts, but not price increases.

• Firms operating in a Sweezy oligopoly maximize profit by producing where

\[ MR_S = MC. \]

• The kinked-shaped marginal revenue curve implies that there exists a range over which changes in MC will not impact the profit-maximizing level of output.

• Therefore, the firm may have no incentive to change price provided that marginal cost remains in a given range.
Cournot Model Environment

• A few firms produce goods that are either perfect substitutes (homogeneous) or imperfect substitutes (differentiated).

• Firms’ control variable is output in contrast to price.

• Each firm believes their rivals will hold output constant if it changes its own output (The output of rivals is viewed as given or “fixed”).

• Barriers to entry exist.
Inverse Demand in a Cournot Duopoly

- Market demand in a homogeneous-product Cournot duopoly is
  \[ P = a - b(Q_1 + Q_2) \]
- Thus, each firm’s marginal revenue depends on the output produced by the other firm. More formally,

  \[ MR_1 = a - bQ_2 - 2bQ_1 \]
  \[ MR_2 = a - bQ_1 - 2bQ_2 \]
Best-Response Function

• Since a firm’s marginal revenue in a homogeneous Cournot oligopoly depends on both its output and its rivals, each firm needs a way to “respond” to rival’s output decisions.

• Firm 1’s best-response (or reaction) function is a schedule summarizing the amount of $Q_1$ firm 1 should produce in order to maximize its profits for each quantity of $Q_2$ produced by firm 2.

• Since the products are substitutes, an increase in firm 2’s output leads to a decrease in the profit-maximizing amount of firm 1’s product.
Best-Response Function for a Cournot Duopoly

• To find a firm’s best-response function, equate its marginal revenue to marginal cost and solve for its output as a function of its rival’s output.

• Firm 1’s best-response function is \((c_1 \text{ is firm 1’s MC})\)

\[ Q_1 = r_1(Q_2) = \frac{a - c_1}{2b} - \frac{1}{2} Q_2 \]

• Firm 2’s best-response function is \((c_2 \text{ is firm 2’s MC})\)

\[ Q_2 = r_2(Q_1) = \frac{a - c_2}{2b} - \frac{1}{2} Q_1 \]
Graph of Firm 1’s
Best-Response Function

\[ Q_1 = r_1(Q_2) = \frac{(a - c_1)}{2b} - 0.5Q_2 \]
Cournot Equilibrium

• Situation where each firm produces the output that maximizes its profits, given the output of rival firms.

• No firm can gain by unilaterally changing its own output to improve its profit.
  • A point where the two firm’s best-response functions intersect.
Graph of Cournot Equilibrium

\[
\frac{a - c_1}{b} \quad \frac{a - c_2}{b}
\]
Summary of Cournot Equilibrium

• The output \( Q_1^* \) maximizes firm 1’s profits, given that firm 2 produces \( Q_2^* \).
• The output \( Q_2^* \) maximizes firm 2’s profits, given that firm 1 produces \( Q_1^* \).
• Neither firm has an incentive to change its output, given the output of the rival.
• Beliefs are consistent:
  • In equilibrium, each firm “thinks” rivals will stick to their current output – and they do!
Firm 1’s Isoprofit Curve

- The combinations of outputs of the two firms that yield firm 1 the same level of profit

\[
\begin{align*}
Q_1 & = \text{firm 1's output} \\
Q_2 & = \text{firm 2's output} \\
M & = \text{market demand} \\
p & = \text{price} \\
\pi_1 & = \text{firm 1's profit}
\end{align*}
\]

Increasing Profits for Firm 1

\[
\begin{align*}
\pi_1 & = $100 \\
\pi_1 & = $200
\end{align*}
\]
Another Look at Cournot Decisions

Firm 1’s best response to $Q_2^*$

$\pi_1 = $100

$\pi_1 = $200
Another Look at Cournot Equilibrium
Impact of Rising Costs on the Cournot Equilibrium

Prior to firm 1’s marginal cost increase:
- Cournot equilibrium $Q_1^*$, $Q_2^*$, $r_1^*$, $r_2^*$

After firm 1’s marginal cost increase:
- Cournot equilibrium $Q_1^{**}$, $Q_2^{**}$, $r_1^{**}$, $r_2$
Collusion Incentives in Cournot Oligopoly
Stackelberg Model Environment

• Few firms serving many consumers.
• Firms produce differentiated or homogeneous products.
• Barriers to entry.
• Firm one is the leader.
  • The leader commits to an output before all other firms.
• Remaining firms are followers.
  • They choose their outputs so as to maximize profits, given the leader’s output.
Stackelberg Equilibrium

Follower’s Profits Decline

Stackelberg Equilibrium
The Algebra of the Stackelberg Model

• Since the follower reacts to the leader’s output, the follower’s output is determined by its reaction function

\[ Q_2 = r_2(Q_1) = \frac{a - c_2}{2b} - 0.5Q_1 \]

• The Stackelberg leader uses this reaction function to determine its profit maximizing output level, which simplifies to

\[ Q_1 = \frac{a + c_2 - 2c_1}{2b} \]
Stackelberg Summary

- Stackelberg model illustrates how commitment can enhance profits in strategic environments.

- Leader produces *more* than the Cournot equilibrium output.
  - Larger market share, higher profits.
  - First-mover advantage.

- Follower produces *less* than the Cournot equilibrium output.
  - Smaller market share, lower profits.
Bertrand Model Environment

• Few firms that sell to many consumers.
• Firms produce identical products at constant marginal cost.
• Each firm independently sets its price in order to maximize profits (price is each firms’ control variable).
• Barriers to entry exist.
• Consumers enjoy
  • Perfect information.
  • Zero transaction costs.
Bertrand Equilibrium

• Firms set $P_1 = P_2 = MC!$ Why?
• Suppose $MC < P_1 < P_2$.
• Firm 1 earns $(P_1 - MC)$ on each unit sold, while firm 2 earns nothing.
• Firm 2 has an incentive to slightly undercut firm 1’s price to capture the entire market.
• Firm 1 then has an incentive to undercut firm 2’s price. This undercutting continues...
• Equilibrium: Each firm charges $P_1 = P_2 = MC$. 
Contestable Markets

• Key Assumptions
  • Producers have access to same technology.
  • Consumers respond quickly to price changes.
  • Existing firms cannot respond quickly to entry by lowering price.
  • Absence of sunk costs.

• Key Implications
  • Threat of entry disciplines firms already in the market.
  • Incumbents have no market power, even if there is only a single incumbent (a monopolist).
Conclusion

• Different oligopoly scenarios give rise to different optimal strategies and different outcomes.

• Your optimal price and output depends on ...
  • Beliefs about the reactions of rivals.
  • Your choice variable (P or Q) and the nature of the product market (differentiated or homogeneous products).
  • Your ability to credibly commit prior to your rivals.
GAME THEORY: INSIDE OLIGOPOLY
Overview

I. Introduction to Game Theory
II. Simultaneous-Move, One-Shot Games
III. Infinitely Repeated Games
IV. Finitely Repeated Games
V. Multistage Games
Game Environments

• Players’ planned decisions are called strategies.
• Payoffs to players are the profits or losses resulting from strategies.
• Order of play is important:
  • Simultaneous-move game: each player makes decisions with knowledge of other players’ decisions.
  • Sequential-move game: one player observes its rival’s move prior to selecting a strategy.
• Frequency of rival interaction
  • One-shot game: game is played once.
  • Repeated game: game is played more than once; either a finite or infinite number of interactions.
Simultaneous-Move, One-Shot Games: Normal Form Game

• A Normal Form Game consists of:
  • Set of players $i \in \{1, 2, ... n\}$ where $n$ is a finite number.
  • Each players strategy set or feasible actions consist of a finite number of strategies.
    • Player 1’s strategies are $S_1 = \{a, b, c, ...\}$.
    • Player 2’s strategies are $S_2 = \{A, B, C, ...\}$.
  • Payoffs.
    • Player 1’s payoff: $\pi_1(a,B) = 11$.
    • Player 2’s payoff: $\pi_2(b,C) = 12$. 
A Normal Form Game

<table>
<thead>
<tr>
<th>Strategy</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>12,11</td>
<td>11,12</td>
<td>14,13</td>
</tr>
<tr>
<td>b</td>
<td>11,10</td>
<td>10,11</td>
<td>12,12</td>
</tr>
<tr>
<td>c</td>
<td>10,15</td>
<td>10,13</td>
<td>13,14</td>
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</tbody>
</table>
Normal Form Game: Scenario Analysis

• Suppose 1 thinks 2 will choose “A”.

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<th>Player 2</th>
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<tbody>
<tr>
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<td>10,15</td>
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</tbody>
</table>
Normal Form Game: Scenario Analysis

- Then 1 should choose “a”.
  - Player 1’s best response to “A” is “a”.

<table>
<thead>
<tr>
<th>Player 1</th>
<th>A</th>
<th>B</th>
<th>C</th>
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</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
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</table>
Normal Form Game: Scenario Analysis

• Suppose 1 thinks 2 will choose “B”.

<table>
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</tbody>
</table>
Normal Form Game: Scenario Analysis

• Then 1 should choose “a”.
  • Player 1’s best response to “B” is “a”.

<table>
<thead>
<tr>
<th>Strategy</th>
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</tbody>
</table>
Normal Form Game
Scenario Analysis

- Similarly, if 1 thinks 2 will choose C....
  - Player 1’s best response to “C” is “a”.

<table>
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Dominant Strategy

• Regardless of whether Player 2 chooses A, B, or C, Player 1 is better off choosing “a”!
• “a” is Player 1’s Dominant Strategy!

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</table>
Dominant Strategy in a Simultaneous-Move, One-Shot Game

• A dominant strategy is a strategy resulting in the highest payoff regardless of the opponent’s action.

• If “a” is a dominant strategy for Player 1 in the previous game, then:
  • \( \pi_1(a,A) > \pi_1(b,A) \geq \pi_1(c,A) \);
  • \( \pi_1(a,B) > \pi_1(b,B) \geq \pi_1(c,B) \);
  • and \( \pi_1(a,C) > \pi_1(b,C) \geq \pi_1(c,C) \).
Putting Yourself in your Rival’s Shoes

• What should player 2 do?
  • 2 has no dominant strategy!
  • But 2 should reason that 1 will play “a”.
  • Therefore 2 should choose “C”.

<table>
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<tr>
<td>c</td>
<td>10,15</td>
<td>10,13</td>
<td>13,14</td>
</tr>
</tbody>
</table>
The Outcome

- This outcome is called a Nash equilibrium:
  - “a” is player 1’s best response to “C”.
  - “C” is player 2’s best response to “a”.

<table>
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<td>10,15</td>
<td>10,13</td>
<td>13,14</td>
</tr>
</tbody>
</table>
Two-Player Nash Equilibrium

• The Nash equilibrium is a condition describing the set of strategies in which no player can improve her payoff by unilaterally changing her own strategy, given the other player’s strategy.

• Formally,
  • \( \pi_1(s_1^*, s_2^*) \geq \pi_1(s_1, s_2^*) \) for all \( s_1 \).
  • \( \pi_1(s_1^*, s_2^*) \geq \pi_1(s_1^*, s_2) \) for all \( s_2 \).
Key Insights

• Look for dominant strategies.
• Put yourself in your rival’s shoes.
A Market-Share Game

• Two managers want to maximize market share: \( i \in \{1, 2\} \).
• Strategies are pricing decisions
  • \( S_1 = \{1, 5, 10\} \).
  • \( S_2 = \{1, 5, 10\} \).
• Simultaneous moves.
• One-shot game.
The Market-Share Game in Normal Form

<table>
<thead>
<tr>
<th>Strategy</th>
<th>P=$10</th>
<th>P=$5</th>
<th>P = $1</th>
</tr>
</thead>
<tbody>
<tr>
<td>P=$10</td>
<td>.5, .5</td>
<td>.2, .8</td>
<td>.1, .9</td>
</tr>
<tr>
<td>P=$5</td>
<td>.8, .2</td>
<td>.5, .5</td>
<td>.2, .8</td>
</tr>
<tr>
<td>P=$1</td>
<td>.9, .1</td>
<td>.8, .2</td>
<td>.5, .5</td>
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</table>
Market-Share Game Equilibrium

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Manager 1</th>
<th>Manager 2</th>
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<tbody>
<tr>
<td>P=$10</td>
<td>P=$10 .5, .5</td>
<td>P=$10 .2, .8</td>
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<tr>
<td>P=$5</td>
<td>P=$5 .8, .2</td>
<td>P=$5 .5, .5</td>
</tr>
<tr>
<td>P=$1</td>
<td>P=$1 .9, .1</td>
<td>P=$1 .8, .2</td>
</tr>
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</table>

Manager 2

Nash Equilibrium
Key Insight:

• Game theory can be used to analyze situations where “payoffs” are non monetary!

• We will, without loss of generality, focus on environments where businesses want to maximize profits.
  • Hence, payoffs are measured in monetary units.
Coordination Games

• In many games, players have competing objectives: One firm gains at the expense of its rivals.
• However, some games result in higher profits by each firm when they “coordinate” decisions.
Examples of Coordination Games

• Industry standards
  • size of floppy disks.
  • size of CDs.

• National standards
  • electric current.
  • traffic laws.
A Coordination Game in Normal Form

<table>
<thead>
<tr>
<th>Strategy</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0,0</td>
<td>0,0</td>
<td>$10,$10</td>
</tr>
<tr>
<td>2</td>
<td>$10,$10</td>
<td>0,0</td>
<td>0,0</td>
</tr>
<tr>
<td>3</td>
<td>0,0</td>
<td>$10,$10</td>
<td>0,0</td>
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</table>
A Coordination Problem: Three Nash Equilibria!

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Strategy</th>
<th>A</th>
<th>B</th>
<th>C</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td>0,0</td>
<td>0,0</td>
<td>$10,$10</td>
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<td>3</td>
<td></td>
<td>0,0</td>
<td>$10, $10</td>
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Key Insights:

• Not all games are games of conflict.
• Communication can help solve coordination problems.
• Sequential moves can help solve coordination problems.
Games With No Pure Strategy Nash Equilibrium

<table>
<thead>
<tr>
<th>Strategy</th>
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<th>Player 2</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>100,-100</td>
</tr>
<tr>
<td>2</td>
<td>100,-100</td>
<td>-100,100</td>
</tr>
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</table>
Strategies for Games With No Pure Strategy Nash Equilibrium

• In games where no pure strategy Nash equilibrium exists, players find it in their interest to engage in mixed (randomized) strategies.
  • This means players will “randomly” select strategies from all available strategies.
An Advertising Game

- Two firms (Kellogg’s & General Mills) managers want to maximize profits.
- Strategies consist of advertising campaigns.
- Simultaneous moves.
  - One-shot interaction.
  - Repeated interaction.
A One-Shot Advertising Game

<table>
<thead>
<tr>
<th>Kellogg's Strategy</th>
<th>None</th>
<th>Moderate</th>
<th>High</th>
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<tr>
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<td>6, 6</td>
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<tr>
<td>High</td>
<td>15, -1</td>
<td>9, 0</td>
<td>2, 2</td>
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General Mills
Equilibrium to the One-Shot Advertising Game

<table>
<thead>
<tr>
<th>Strategy</th>
<th>General Mills</th>
<th>Kellogg's</th>
</tr>
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<tbody>
<tr>
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<td>None</td>
<td>Moderate</td>
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<td></td>
<td>12,12</td>
<td>1, 20</td>
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<tr>
<td>Moderate</td>
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<td>High</td>
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<td></td>
<td>20, 1</td>
<td>6, 6</td>
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<tr>
<td>High</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>15, -1</td>
<td>9, 0</td>
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</table>

Nash Equilibrium: (High, High)
Can collusion work if the game is repeated 2 times?

<table>
<thead>
<tr>
<th>Strategy</th>
<th>General Mills</th>
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<tbody>
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<td>Moderate</td>
<td>20, 1</td>
</tr>
<tr>
<td>High</td>
<td>15, -1</td>
</tr>
<tr>
<td></td>
<td>1, 20</td>
</tr>
<tr>
<td></td>
<td>6, 6</td>
</tr>
<tr>
<td></td>
<td>9, 0</td>
</tr>
<tr>
<td></td>
<td>-1, 15</td>
</tr>
<tr>
<td></td>
<td>0, 9</td>
</tr>
<tr>
<td></td>
<td>2, 2</td>
</tr>
</tbody>
</table>
No (by backwards induction).

• In period 2, the game is a one-shot game, so equilibrium entails High Advertising in the last period.
• This means period 1 is “really” the last period, since everyone knows what will happen in period 2.
• Equilibrium entails High Advertising by each firm in both periods.
• The same holds true if we repeat the game any known, finite number of times.
Can collusion work if firms play the game each year, forever?

• Consider the following “trigger strategy” by each firm:
  • “Don’t advertise, provided the rival has not advertised in the past. If the rival ever advertises, “punish” it by engaging in a high level of advertising forever after.”
• In effect, each firm agrees to “cooperate” so long as the rival hasn’t “cheated” in the past. “Cheating” triggers punishment in all future periods.
Suppose General Mills adopts this trigger strategy. Kellogg’s profits?

\[
\Pi_{\text{Cooperate}} = 12 + \frac{12}{(1+i)} + \frac{12}{(1+i)^2} + \frac{12}{(1+i)^3} + \ldots \\
= 12 + \frac{12}{i} \\
\Pi_{\text{Cheat}} = 20 + \frac{2}{(1+i)} + \frac{2}{(1+i)^2} + \frac{2}{(1+i)^3} + \ldots \\
= 20 + \frac{2}{i}
\]

Value of a perpetuity of $12 paid at the end of every year

<table>
<thead>
<tr>
<th>Strategy</th>
<th>None</th>
<th>Moderate</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>12, 12</td>
<td>1, 20</td>
<td>-1, 15</td>
</tr>
<tr>
<td>Moderate</td>
<td>20, 1</td>
<td>6, 6</td>
<td>0, 9</td>
</tr>
<tr>
<td>High</td>
<td>15, -1</td>
<td>9, 0</td>
<td>2, 2</td>
</tr>
</tbody>
</table>
Kellogg’s Gain to Cheating:

- $\Pi_{\text{Cheat}} - \Pi_{\text{Cooperate}} = 20 + \frac{2}{i} - (12 + \frac{12}{i}) = 8 - \frac{10}{i}$
  - Suppose $i = 0.05$
  - $\Pi_{\text{Cheat}} - \Pi_{\text{Cooperate}} = 8 - \frac{10}{0.05} = 8 - 200 = -192$

- It doesn’t pay to deviate.
  - Collusion is a Nash equilibrium in the infinitely repeated game!

<table>
<thead>
<tr>
<th>Kellogg’s</th>
<th>General Mills</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>None</td>
<td>12,12</td>
</tr>
<tr>
<td>Moderate</td>
<td>20, 1</td>
</tr>
<tr>
<td>High</td>
<td>15, -1</td>
</tr>
</tbody>
</table>
Benefits & Costs of Cheating

• $\Pi_{\text{Cheat}} - \Pi_{\text{Cooperate}} = 8 - \frac{10}{i}$
  • $8 = \text{Immediate Benefit (20 - 12 today)}$
  • $\frac{10}{i} = \text{PV of Future Cost (12 - 2 forever after)}$

• If Immediate Benefit - PV of Future Cost > 0
  • Pays to “cheat”.

• If Immediate Benefit - PV of Future Cost ≤ 0
  • Doesn’t pay to “cheat”.

<table>
<thead>
<tr>
<th>General Mills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>None</td>
</tr>
<tr>
<td>Moderate</td>
</tr>
<tr>
<td>High</td>
</tr>
</tbody>
</table>

Strategy: None, Moderate, High

Kellogg’s
Key Insight

• Collusion can be sustained as a Nash equilibrium when there is no certain “end” to a game.

• Doing so requires:
  • Ability to monitor actions of rivals.
  • Ability (and reputation for) punishing defectors.
  • Low interest rate.
  • High probability of future interaction.
Real World Examples of Collusion

• Garbage Collection Industry
• OPEC
• NASDAQ
• Airlines
• Lysine Market
Normal-Form Bertrand Game

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td><strong>Low Price</strong></td>
<td><strong>High Price</strong></td>
</tr>
<tr>
<td>Low Price</td>
<td>0,0</td>
<td>20,-1</td>
</tr>
<tr>
<td>High Price</td>
<td>-1, 20</td>
<td>15, 15</td>
</tr>
</tbody>
</table>

Firm 1 and Firm 2 are firms competing in a Bertrand game, with strategies being low price or high price.
One-Shot Bertrand (Nash) Equilibrium

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td><strong>Low Price</strong></td>
</tr>
<tr>
<td><strong>Low Price</strong></td>
<td>0,0</td>
</tr>
<tr>
<td><strong>High Price</strong></td>
<td>-1, 20</td>
</tr>
</tbody>
</table>
Potential Repeated Game
Equilibrium Outcome

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Price</td>
<td>0,0</td>
<td>20,-1</td>
</tr>
<tr>
<td>High Price</td>
<td>-1, 20</td>
<td>15, 15</td>
</tr>
</tbody>
</table>
Simultaneous-Move Bargaining

• Management and a union are negotiating a wage increase.
• Strategies are wage offers & wage demands.
• Successful negotiations lead to $600 million in surplus, which must be split among the parties.
• Failure to reach an agreement results in a loss to the firm of $100 million and a union loss of $3 million.
• Simultaneous moves, and time permits only one-shot at making a deal.
The Bargaining Game in Normal Form

<table>
<thead>
<tr>
<th>Strategy</th>
<th>W = $10</th>
<th>W = $5</th>
<th>W = $1</th>
</tr>
</thead>
<tbody>
<tr>
<td>W = $10</td>
<td>100, 500</td>
<td>-100, -3</td>
<td>-100, -3</td>
</tr>
<tr>
<td>W = $5</td>
<td>-100, -3</td>
<td>300, 300</td>
<td>-100, -3</td>
</tr>
<tr>
<td>W = $1</td>
<td>-100, -3</td>
<td>-100, -3</td>
<td>500, 100</td>
</tr>
</tbody>
</table>
Three Nash Equilibria!

<table>
<thead>
<tr>
<th>Strategy</th>
<th>W = $10</th>
<th>W = $5</th>
<th>W = $1</th>
</tr>
</thead>
<tbody>
<tr>
<td>W = $10</td>
<td>100, 500</td>
<td>-100, -3</td>
<td>-100, -3</td>
</tr>
<tr>
<td>W = $5</td>
<td>-100, -3</td>
<td>300, 300</td>
<td>-100, -3</td>
</tr>
<tr>
<td>W = $1</td>
<td>-100, -3</td>
<td>-100, -3</td>
<td>500, 100</td>
</tr>
</tbody>
</table>
Fairness: The “Natural” Focal Point

<table>
<thead>
<tr>
<th>Strategy</th>
<th>( W = $10 )</th>
<th>( W = $5 )</th>
<th>( W = $1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W = $10 )</td>
<td>100, 500</td>
<td>-100, -3</td>
<td>-100, -3</td>
</tr>
<tr>
<td>( W = $5 )</td>
<td>-100, -3</td>
<td>300, 300</td>
<td>-100, -3</td>
</tr>
<tr>
<td>( W = $1 )</td>
<td>-100, -3</td>
<td>-100, -3</td>
<td>500, 100</td>
</tr>
</tbody>
</table>
Lessons in Simultaneous Bargaining

• Simultaneous-move bargaining results in a coordination problem.
• Experiments suggest, in the absence of any “history,” real players typically coordinate on the “fair outcome.”
• When there is a “bargaining history,” other outcomes may prevail.
Single-Offer Bargaining

• Now suppose the game is sequential in nature, and management gets to make the union a “take-it-or-leave-it” offer.

• Analysis Tool: Write the game in extensive form
  • Summarize the players.
  • Their potential actions.
  • Their information at each decision point.
  • Sequence of moves.
  • Each player’s payoff.
Step 1: Management’s Move
Step 2: Add the Union’s Move

Firm

Union

Accept

Reject

Union

Accept

Reject

Union

Accept

Reject
Step 3: Add the Payoffs

- **Firm:**
  - 10
  - 5
  - 1

- **Union:**
  - **Accept:** 100, 500
  - **Reject:** -100, -3

- **Union:**
  - **Accept:** 300, 300
  - **Reject:** -100, -3

- **Union:**
  - **Accept:** 500, 100
  - **Reject:** -100, -3
The Game in Extensive Form

Firm

Union
Accept: 100, 500
Reject: -100, -3

Union
Accept: 300, 300
Reject: -100, -3

Union
Accept: 500, 100
Reject: -100, -3
Step 4: Identify the Firm’s Feasible Strategies

- Management has one information set and thus three feasible strategies:
  - Offer $10.
  - Offer $5.
  - Offer $1.
Step 5: Identify the Union’s Feasible Strategies

- The Union has three information set and thus eight feasible strategies ($2^3=8$):
  - Accept $10$, Accept $5$, Accept $1$
  - Accept $10$, Accept $5$, Reject $1$
  - Accept $10$, Reject $5$, Accept $1$
  - Accept $10$, Reject $5$, Reject $1$
  - Reject $10$, Accept $5$, Accept $1$
  - Reject $10$, Accept $5$, Reject $1$
  - Reject $10$, Reject $5$, Accept $1$
  - Reject $10$, Reject $5$, Reject $1$
Step 6: Identify Nash Equilibrium Outcomes

- Outcomes such that neither the firm nor the union has an incentive to change its strategy, given the strategy of the other.
## Finding Nash Equilibrium Outcomes

<table>
<thead>
<tr>
<th>Union’s Strategy</th>
<th>Firm's Best Response</th>
<th>Mutual Best Response?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accept $10, Accept $5, Accept $1</td>
<td>$1</td>
<td>Yes</td>
</tr>
<tr>
<td>Accept $10, Accept $5, Reject $1</td>
<td>$5</td>
<td>Yes</td>
</tr>
<tr>
<td>Accept $10, Reject $5, Accept $1</td>
<td>$1</td>
<td>Yes</td>
</tr>
<tr>
<td>Reject $10, Accept $5, Accept $1</td>
<td>$1</td>
<td>Yes</td>
</tr>
<tr>
<td>Accept $10, Reject $5, Reject $1</td>
<td>$10</td>
<td>Yes</td>
</tr>
<tr>
<td>Reject $10, Accept $5, Reject $1</td>
<td>$5</td>
<td>Yes</td>
</tr>
<tr>
<td>Reject $10, Reject $5, Accept $1</td>
<td>$1</td>
<td>Yes</td>
</tr>
<tr>
<td>Reject $10, Reject $5, Reject $1</td>
<td>$10, $5, $1</td>
<td>No</td>
</tr>
</tbody>
</table>
Step 7: Find the Subgame Perfect Nash Equilibrium Outcomes

• Outcomes where no player has an incentive to change its strategy, given the strategy of the rival, and

• The outcomes are based on “credible actions;” that is, they are not the result of “empty threats” by the rival.
## Checking for Credible Actions

<table>
<thead>
<tr>
<th>Union's Strategy</th>
<th>Are all Actions Credible?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accept $10, Accept $5, Accept $1</td>
<td>Yes</td>
</tr>
<tr>
<td>Accept $10, Accept $5, Reject $1</td>
<td>No</td>
</tr>
<tr>
<td>Accept $10, Reject $5, Accept $1</td>
<td>No</td>
</tr>
<tr>
<td>Reject $10, Accept $5, Accept $1</td>
<td>No</td>
</tr>
<tr>
<td>Accept $10, Reject $5, Reject $1</td>
<td>No</td>
</tr>
<tr>
<td>Reject $10, Accept $5, Reject $1</td>
<td>No</td>
</tr>
<tr>
<td>Reject $10, Reject $5, Accept $1</td>
<td>No</td>
</tr>
<tr>
<td>Reject $10, Reject $5, Reject $1</td>
<td>No</td>
</tr>
</tbody>
</table>
The “Credible” Union Strategy

<table>
<thead>
<tr>
<th>Union's Strategy</th>
<th>Are all Actions Credible?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accept $10, Accept $5, Accept $1</td>
<td>Yes</td>
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<tr>
<td>Accept $10, Accept $5, Reject $1</td>
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</tr>
<tr>
<td>Accept $10, Reject $5, Accept $1</td>
<td>No</td>
</tr>
<tr>
<td>Reject $10, Accept $5, Accept $1</td>
<td>No</td>
</tr>
<tr>
<td>Accept $10, Reject $5, Reject $1</td>
<td>No</td>
</tr>
<tr>
<td>Reject $10, Accept $5, Reject $1</td>
<td>No</td>
</tr>
<tr>
<td>Reject $10, Reject $5, Accept $1</td>
<td>No</td>
</tr>
<tr>
<td>Reject $10, Reject $5, Reject $1</td>
<td>No</td>
</tr>
</tbody>
</table>
## Finding Subgame Perfect Nash Equilibrium Strategies

<table>
<thead>
<tr>
<th>Union's Strategy</th>
<th>Firm's Best Response</th>
<th>Mutual Best Response?</th>
</tr>
</thead>
<tbody>
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<td>Accept $10, Accept $5, Accept $1</td>
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<td>Yes</td>
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<td>Reject $10, Accept $5, Accept $1</td>
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<td>$1</td>
<td>Yes</td>
</tr>
<tr>
<td>Reject $10, Reject $5, Reject $1</td>
<td>$10, $5, $1</td>
<td>No</td>
</tr>
</tbody>
</table>

- **Nash and Credible**
- **Nash Only**
- **Neither Nash Nor Credible**
To Summarize:

• We have identified many combinations of Nash equilibrium strategies.

• In all but one the union does something that isn’t in its self interest (and thus entail threats that are not credible).

• Graphically:
There are 3 Nash Equilibrium Outcomes!

10, 500
-100, -3

300, 300
-100, -3

500, 100
-100, -3
Only 1 Subgame-Perfect Nash Equilibrium Outcome!

Firm

Union

Accept

Reject

10, 500

-100, -3

Union

Accept

Reject

300, 300

-100, -3

Union

Accept

Reject

500, 100

-100, -3
Bargaining Re-Cap

- In take-it-or-leave-it bargaining, there is a first-mover advantage.
- Management can gain by making a take-it-or-leave-it offer to the union. But...
- Management should be careful; real world evidence suggests that people sometimes reject offers on the basis of “principle” instead of cash considerations.
Pricing to Prevent Entry: An Application of Game Theory

• Two firms: an incumbent and potential entrant.
• Potential entrant’s strategies:
  • Enter.
  • Stay Out.
• Incumbent’s strategies:
  • {if enter, play hard}.
  • {if enter, play soft}.
  • {if stay out, play hard}.
  • {if stay out, play soft}.
• Move Sequence:
  • Entrant moves first. Incumbent observes entrant’s action and selects an action.
The Pricing to Prevent Entry Game in Extensive Form

Entrant

Enter

Out

Incumbent

Hard

-1, 1

Soft

5, 5

0, 10
Identify Nash and Subgame Perfect Equilibria

```
Entrant
  \--------
  |        |
  |        |
  Enter   Incumbent
  \--------
    |       |
    |       |
    Out    Hard
           \--
           -1, 1

    |       |
    |       |
    Soft   5, 5
    \--
      0, 10
```
Two Nash Equilibria

Nash Equilibria Strategies \{\text{player 1; player 2}\}:
\{\text{enter; If enter, play soft}\}
\{\text{stay out; If enter, play hard}\}
One Subgame Perfect Equilibrium

Subgame Perfect Equilibrium Strategy:
{enter; If enter, play soft}
Insights

• Establishing a reputation for being unkind to entrants can enhance long-term profits.

• It is costly to do so in the short-term, so much so that it isn’t optimal to do so in a one-shot game.